Mathematics GCSE

Course plan

This plan shows the structure of the course and gives an outline of the contents.

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Sample of the GCSE Mathematics Course from Section 1

This sample material is in pdf format to give you an idea of the content. However, Sections 1-3 of the course are set up with a combination of text, examples, videos and online questions, in order to give step-by-step guidance at the crucial early stages of the course, so you can be sure to learn the fundamental principles of mathematics

Topic 1

Working with numbers

Why learn about working with numbers?

A large part of mathematics is concerned with number. You will come across lots of different types of number during this course: whole numbers, negative numbers, decimals and fractions are some of them.

In this topic we will look at the familiar counting numbers, that is the positive whole numbers (including zero) – these are also known as the positive integers – and learn to work with them without the aid of a calculator.

So why learn about numbers?

- Being able to do calculations quickly and without a calculator is a useful skill in countless everyday situations.
- Practice at this will mean you can do many calculations more quickly than you could on a calculator or your phone.
- You will gain an intuitive understanding of number and how calculations work, which will mean you can more readily notice

when mistakes have been made, for example on your calculator, at a restaurant or on a receipt.

You will probably need 3 hours to complete this topic. However, much of this topic may be familiar and straightforward to you, so you may be able to work through it more quickly.

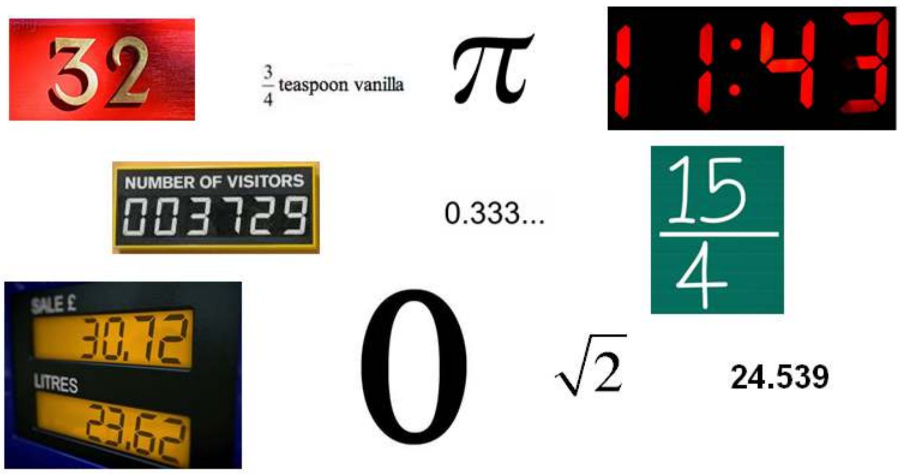
Objectives

When you have finished this topic, you should be able to:

- recall the times tables up to 12 x 12
- find the complement to 10 of any single-digit number and the complement to 100 of any two-digit number
- add any two whole numbers without a calculator
- subtract any two whole numbers without a calculator
- multiply any number by a multiple of 10, e.g. 43 × 20
- multiply any number by a single digit without a calculator, e.g. 473 × 7
- multiply any two whole numbers together without a calculator, e.g. 37 × 64 and 45 × 271
- divide any number by a single digit, e.g. 873 ÷ 6 and appreciate that there may be a remainder
- divide any whole number by another whole number, e.g. 8734 ÷ 32
- define the words integer, sum, difference, product and operation
- apply the skills learned to solve real-world problems.

Introduction

There are many different kinds of number:



Here we shall focus on the group of numbers called the **positive integers** (including zero). An **integer** is a whole number, so these are just the positive whole numbers, or the counting numbers. Let's start by looking at some positive integers and what they're made up of.

Integers, digits and place values

What are integers made up of?

The numbers 3, 45 and 100 are of different lengths, so we say that they have a different number of **digits**.

For example, the number 45 is made up of two digits. The first digit is 4 and the second digit is 5. We say 45 is a two-digit number.

Here, 4 and 5 stand for different things because they are at different places in the number:

100	**10**	**1**
	4	5

The 4 is in the tens column and means 40, or 4 lots of 10.

The 5 is in the ones column and just means 5, or 5 lots of 1, which we could write as 5 × 1.

When we add another digit to the left-hand end to make, say 345,

100	**10**	**1**
3	4	5

we know this is three hundred and forty-five and so the 3 means three hundred or 3 lots of 100.

When we add a fourth digit, e.g.
6 345, the 6 means six thousand or 6 lots of 1 000, which we could write as 6 × 1 000.

When we add a fifth digit, e.g.
76 345, the 7 means seventy thousand or 7 lots of 10 000, which we could write as 7 × 10 000.

When we add a sixth digit, e.g.
276 345, the 2 means two hundred thousand or 2 lots of 100 000, which we could write as 2 × 100 000.

Finally, when we add a seventh digit, e.g.
8 276 345, the 8 means eight million or 8 lots of 1 000 000, which we could write as 8 × 1 000 000.

1 000 000	**100 000**	**10 000**	**1 000**	**100**	**10**	**1**
8	2	7	6	3	4	5

Note that quite often, to make the digits easy to read and to see more quickly how big the number is, we separate every three numbers, starting from the right-hand side, with a space (or in some places you may see it with a comma).

One thing we need to be able to do is to give the value of any digit in a number – this is called its **place value** because its value depends on the place that it occupies in the number. Let's have a look at an example.

Example: In the number 73 894 give the place value of the digit '3' and the digit '9'.

Solution:

10 000	1 000	100	10	1
7	3	8	9	4

Counting from the right, the fourth figure gives the number of *thousands*. So the 3 means 3 000.
The 9 is in the *tens* place, so the 9 means 90.

Key point

A number is made up of digits e.g. 45 is a two-digit number the first digit is 4 and the second digit is 5.

Key point

Each digit has a place value corresponding to its position in the number. For example, the place value of 3 in the number 1 345 is 300.

Activity 1

1 In the number 203 545, select the correct place value for the following digits:

a) 4 b) 2 c) 3

2 In the number 4 305 719, write down the correct place value in words for the following digits:

a) 7 b) 9 c) 4 d) 3

The multiplication table

There are some skills that make working with numbers very much easier. One of these is knowing the multiplication table – the more familiar you are with it, the quicker and more accurate you will be in your calculations.

At first you may find it useful as a reference, but hopefully in time you will not need to refer to it. You should be able to recall all of the multiplications in this table.

Activity 2

Fill in the missing numbers in Table 1 below.

Table 1

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15							
4	4	8	12	16								
5	5	10	15									
6	6	12										
7	7											
8	8	16										
9	9	18	27									
10	10	20	30	40								
11	11	22	33	44	55							
12	12	24	36	48	60	72						

The following statements refer just to the numbers in the completed multiplication table. Are they true or false?

(a) All the multiples of 10 (the numbers in the 10 time table) have 0 for their ‘ones’ digit. i.e. they end in 0.

(b) All the multiples of 5 (the numbers in the 5 times table) end in a 5.

(c) If you take any number in the 7 times table (except 77) and add together its digits, you get 7.

(d) If you take any number in the 9 times table (except 99) and add together its digits, you get 9.

(e) If you take any number in the 11 times table and add together its digits, you get an even number.

(f) If you take any number in the 12 times table and add together its digits, you get an even number.

(g) The ones digit of all the numbers on the diagonal from top left to bottom right is either a 1, 4, 5, 6 or 9.

(h) The 'ones' digit of all the numbers in the 2 times table is either a 0, 2, 4, 6 or 8.

(i) The ones digit of all the numbers in the 3 times table is either a 0, 3, 6 or 9.

(j) The number in the 6th row, 4th column is the same as the number in the 4th row, 6th column.

Complements to 10 and to 100

Another very useful skill is being able to see which pairs of positive integers (positive whole numbers) add up to 10 and which add up to 100.

Key point

Pairs of numbers that add up to 10 are called COMPLEMENTS to 10.e.g. the complement to 10 of 3 is 7.

Key point

Pairs of numbers that add up to 100 are called COMPLEMENTS to 100. e.g. the complement to 100 of 36 is 64.

Activity 3

1 Match the numbers which form complements to 10 – the first has been done for you:

1	2
3	6
4	9
5	7
8	5

2 Without using a calculator, find the complement to 100 of the following:

(a)	13	(b)	45	(c)	51	(d)	34
(e)	66	(f)	91	(g)	28	(h)	7

Can you find the complement to 100 of some more integers between 0 and 100?

Look at the pairs of numbers that form complements to 100. What do you notice about the **sum** of the 'tens' digits?

What do you notice about the sum of the 'ones' digits?

Can you explain why you always get these numbers?

Let's look at 16 and 84 as an example.

You should have noticed that the 'ones' digits are always complements to 10. (Here, they are 6 and 4).

The 'tens' digits must therefore always add up to 9, to make 90 (here they are 1 and 8) so that the total is 100.

This is a quick way of finding complements.

Keep practising by giving yourself two-digit numbers, until you can find complements to 100 quickly.

Addition and subtraction

The instruction to do something to some numbers is called an **operation**, and over the next few sections we shall look at several operations on numbers. The four main operations on numbers are addition, subtraction, multiplication and division.

When you do calculations, it is often helpful to keep the digits of the same value lined up in a column.

Here is 764 written in columns:

	H	T	O
	7	0	0
+		6	0
+			4

H stands for hundreds
T stands for tens
O stands for ones

This will be worth bearing in mind over the next few sections.

First, we are going to look at two basic operations, adding and subtracting.

Adding numbers

There are various key words that indicate you should add numbers: sum, total and plus being the most common. You may be able to add to this list.

Let's look at one method of adding whole numbers without a calculator.

Suppose we want to add a number to 764, for example to calculate 764 + 225. To do this, we first arrange the numbers so that the digits line up with each other in their appropriate place:

	H	T	O
	7	6	4
+	2	2	5

We then add each column in turn, beginning with the ones column, writing the answer underneath as we go.

$$
\begin{array}{cccc}
 & \mathbf{H} & \mathbf{T} & \mathbf{O} \\
 & 7 & 6 & 4 \\
+ & 2 & 2 & 5 \\
\hline
 & 9 & 8 & 9
\end{array}
$$

and so we have found that 764 + 225 = 989.

If, when we add the numbers in a particular column, they come to more than nine, we carry the extra tens unit to the next column. Let’s do a slightly altered calculation to illustrate this: 764 + 228.

$$
\begin{array}{cccc}
 & \mathbf{H} & \mathbf{T} & \mathbf{O} \\
 & 7 & 6 & 4 \\
+ & 2 & {}_{1}2 & 8 \\
\hline
 & 9 & 9 & 2
\end{array}
$$

When we add the 4 and the 8 in the ones column to give 12, we put the 2 where it belongs in the ones column and the 1 is carried to the tens column. Now, when we come to the tens column, we have to add this extra 1 in with the 6 + 2 to give 9 altogether. And finally we can just add the 7 and 2 to get 9.

So 764 + 228 = 992.

Here is another example.

$$
\begin{array}{ccccc}
 & & \mathbf{H} & \mathbf{T} & \mathbf{U} \\
 & & 8 & 5 & 9 \\
+ & & {}_{1}5 & {}_{1}7 & 4 \\
\hline
 & 1 & 4 & 3 & 3
\end{array}
$$

Here we have 9 + 4 which gives 13, so we leave the 3 in the ones column and the 1 is carried over to the tens column.

5 + 7 gives 12 plus the extra 1 is 13, so the 3 goes underneath the numbers we have just added and 1 is carried over to the hundreds column.

8 + 5 gives 13 plus the extra 1 is 14, so the 4 goes underneath and the 1 is carried over.

This time, there are no other numbers to add and so we just put the 1 in the next column along – the thousands column.

So our calculation is 859 + 574 = 1433.

We can check this on our calculator, but be careful not to rely on this – there is frequently an addition calculation to carry out on the non-calculator exam.

This method is probably one you have met before, and it always works. However, you might have your own method. It is perfectly fine to use this, as long as it works in any circumstances and you can show your working in the exam. You can always check with your tutor that your method is okay to use.

Activity 4

Find the following sums without a calculator. Make sure you write down your calculations, as there will be marks for this in the exam.

(a) 473 + 126 (b) 532 + 457 (c) 578 + 216

(d) 684 + 259 (e) 749 + 436 (f) 1567 + 583

(g) Two schools are to merge to form one large academy. The first school has 674 pupils and 38 teachers. The second has 553 pupils and 34 teachers.

How many pupils will the combined academy have?

(h) Kate and Sami are arranging their wedding. Kate wants to invite 118 guests and Sami wants to invite 67 guests.

How many quests will they have altogether?

Subtracting two numbers

Now let's look at subtracting whole numbers. There are various key words that indicate you should subtract numbers: **difference**, less than, take away and minus being the most common. Can you add to this list?

Suppose we wish to do the subtraction 650 – 75.

Again, we set the numbers out in their appropriate columns:

	H	T	O
	6	5	0
–		7	5

Remember that the value of each digit depends on the column it is in; hundreds, tens or ones.

Again we work from the 'ones' column and go left. But the difference is that this time we *subtract* the bottom number from the top number, and if the number we are taking away is bigger than the number we're taking it away from, we have to borrow 1 instead of carrying 1. This reduces the next number in the top row by 1. Let's look at how to do this.

$$\begin{array}{cccc} & H & T & O \\ & 6 & {}^{4}\cancel{5} & {}^{1}0 \\ - & & 7 & 5 \\ \hline & & & 5 \end{array}$$

Step 1 We cannot take 5 units from 0, so move a 10 from the tens column to the ones column. So the 5 in the tens column becomes 4 and we have 10 ones in the ones column. Now you can take 5 from 10 to get 5.

$$\begin{array}{cccc} & H & T & O \\ & {}^{5}\cancel{6} & {}^{14}\cancel{5} & {}^{1}0 \\ - & & 7 & 5 \\ \hline & 5 & 7 & 5 \end{array}$$

Step 2 You cannot take 7 tens from 4 tens, so move a 100 from the hundreds column to the tens column. The 6 in the hundreds column becomes 5 and we have 14 tens in the tens column. Now we can take 7 tens from 14 tens, leaving 7 tens.

Step 3 Finally, nothing from 5 hundreds leaves 5 in the hundreds column.

So our answer is that 650 – 75 = 575.

The principle is, whenever you cannot subtract 'bottom number' from 'top number', move a 10 (or 100 or 1 000) from the column to the left.

Here's another example. Let's work out 6 004 – 845.

The same principle applies, but this time the moving has to extend over several columns, because of the 0s in the tens and hundreds.

Again, remember you are subtracting the bottom number (845) from the top one (6 004).

	Th	H	T	O
	6	0	0	4
–		8	4	5
	$^{5}\cancel{6}$	$^{1}0$	0	4
–		8	4	5
	$^{5}\cancel{6}$	$^{9}\cancel{10}$	10	4
–		8	4	5
	$^{5}\cancel{6}$	$^{9}\cancel{10}$	$^{9}\cancel{10}$	$^{1}4$
–		8	4	5
	5	1	5	9

Step 1 We cannot take 5 units from 4 so we go to move a ten from the tens column. However there aren't any. So we move on to try to move 100 from the hundreds column, but there aren't any of these either. So instead we must start by moving 1 000 from the 6 000. This gives us 10 in the hundreds column (because 1 000 is 10 hundreds).

Step 2 We can now move 1 hundred from these (leaving 900 behind), to give 10 tens …

Step 3 … and then we can move one of these to give 9 tens and 14 ones.

Step 4 Now subtract 5 from 14 to get 9 ones subtract 40 from 90 to get 5 tens, 800 from 900 to get 1 hundred and nothing from 5000 to leave 5 thousands.

So our answer is that 6 004 – 845 = 5 159.

You can always check your answer using addition.

Again, this method is probably one you have met before and you might have your own method, which it is perfectly fine to use. Just make sure that it works in any circumstances and that you can show your working in the exam. You can always check with your tutor that your method is okay to use.

Activity 5

Make sure you write down your calculations, as there will be marks for this in the exam.

(a) Find the difference between 672 and 511.

(b) Find the difference between 957 and 428.

(c) What is 369 less than 600?

(d) Subtract 689 from 750.

(e) Subtract 254 from 1 357.

(f) A small company has ordered 6 750 flyers to leaflet the local area. They have handed out 327 at the local shops. There are 5 000 houses in the local area.

How many leaflets do they have left to post door-to-door?

(g) A builder has given you a quote for some work as £2142. This was calculated as £622 for materials, £170 for skip hire and £1350 for labour. As an incentive to get you to take them on, they have offered to knock £250 off the price.

What is the new price?

(h) Callum fills up his car with petrol each week and keeps a record of how many miles he has driven. Last week the mileage on the clock was 77 893. This week the mileage on the clock is 78 129.

How many miles has he driven in the last week?

Multiplication

We are now going to look at two more operations that we can perform on two numbers. The first of these is multiplication.

Before we start, we must appreciate an important fact about multiplication: the order in which we do it does not matter. You saw this when we looked at the multiplication table. So, for example, 34 × 20 is the same calculation as 20 × 34. It doesn't matter which number is put first – in both cases they give the answer 680.

We can see why this must be true from a diagram:

$$2 \times 3 = 6 = 3 \times 2$$

It's easy to see here that 2 lots of 3 and 3 lots of 2 give the same value: 6.

It is the same even if there are three or more numbers:

$$3 \times 12 \times 10$$

is the same as

$$10 \times 3 \times 12$$

or

$$10 \times 12 \times 3$$

and any other arrangement of these three numbers: they all give the same result: 360.

When we have numbers multiplied together such as 2 x 3 we call it a **product.**

We've already looked at more basic multiplication, with the multiplication table. So we want to look at multiplying by bigger numbers. Very helpful here will be knowing how to multiply by 10, 100, 1000, etc.

Multiplying by 10, 100, 1000...

We need to be able to work out some multiplication problems without the help of a calculator. To see how we can do this, we are going to start with something quite easy – multiplying a number by 10.

Multiplying by 10

When we looked at the multiplication table, we saw that all the multiples of 10 ended in a zero. For example,

$$2 \times 10 = 20, \quad 7 \times 10 = 70, \quad \text{etc.}$$

This is because multiplying 7 ones by 10 will give us 7 tens and 0 ones:

Hundreds	Tens	Ones
		7
	7	0

× 10

We can see that the 7 moves one place to the left into the tens column and we put a 0 in the ones column, since there are no ones.

We can guess that the same happens when we multiply any number by 10, and this is quite right. For example:

$$147 \times 10 = 1470$$

Thousands	Hundreds	Tens	Ones
	1	4	7
1	4	7	0

× 10

Multiplying by 100

What about multiplying by 100? Well this is just multiplying by 10 x 10, that is to say multiplying by 10 twice. That means the digits need to move two columns to the left, and the empty columns are filled with two 0s:

Thousands	Hundreds	Tens	Ones
		5	6
	5	6	0
5	6	0	0

× 10

× 10

× 100

and so

$$56 \text{ x } 100 = 5600.$$

Multiplying by 1000

You can probably guess that to multiply by 1000 (which is 10 x 10 x 10) we need to shift everything three columns to the left and fill in the blank columns with zeros. For example, doing 23 x 1000 we get:

equation reference goes hereTen Thousands	Thousands	Hundreds	Tens	Ones
			2	3
2	3	0	0	0

× 1000

and so

$$23 \text{ x } 1000 = 23\,000$$

Clearly the pattern will continue, and we can see that, similarly,

$$724 \text{ x } 1000 = 724\,000$$

$$36 \text{ x } 10\,000 = 360\,000$$

$$441 \text{ x } 100\,000 = 44\,100\,000$$

etc.

Multiplying by multiples of 10, 100, 1000…

What about if we want to multiply a number by 20? Or by 700? The answer is that we do this in stages. Suppose we want to calculate

$$12 \text{ x } 20$$

We, first of all, break down the 20 and write it as

$$2 \times 10$$

and then use the fact that to multiply three numbers together, we could first of all multiply the first two and then the result of this by the third number. Here are some examples.

Example

$$\begin{aligned} 12 \times 20 &= 12 \times 2 \times 10 \\ &= 12 \times 2 \times 10 \\ &= 24 \times 10 \\ &= 240 \end{aligned}$$

Example

$$8 \times 700 = 8 \times 7 \times 100$$
$$= 8 \times 7 \times 100$$
$$= 56 \times 100$$
$$= 5600$$

The same method works when both numbers are multiples of 10:

Example

$$20 \times 700 = 2 \times 10 \times 7 \times 100$$
$$= 2 \times 7 \times 10 \times 100$$
$$= 14 \times 1000$$
$$= 14\,000$$

Example

$$110 \times 80 = 11 \times 10 \times 8 \times 10$$
$$= 11 \times 8 \times 10 \times 10$$
$$= 88 \times 100$$
$$= 8\,800$$

You may find it easier and quicker to do these in your head rather than write the steps out, and that is perfectly fine.

Activity 6

Find the following products (remember that a product is just numbers multiplied together) without using a calculator.

(a) 9×10	(b) 98×10	(c) 32×100	(d) 11×30
(e) 9×700	(f) 2×40	(g) 7×30	(h) 8×5000
(i) 400×20	(j) 600×800	(k) 30×120	(l) 110×110

Long multiplication

We can use another method of splitting up to help us work out products such as

$$76 \times 8 \quad \text{and} \quad 236 \times 5$$

Method 1

How might we calculate 76 x 8?

What we shall do is use the fact that 76 is the same as 70 + 6 and then multiply each of these bits by 8. So 76 × 8 = 70 × 8 + 6 × 8.

Now 70 × 8 = 7 × 8 × 10 = 56 × 10 = 560.

We also have 6 × 8 = 48.

Therefore we have 76 × 8 = 560 + 48.

And so we have that 76 x 8 = 608.

Example. Calculate 891 × 9.

Use the fact that 891 is the same as 800 + 90 + 1 and then multiply each of these bits by 9.

So 891 × 9 becomes 800 × 9 + 90 × 9 + 1 × 9.

Now 800 × 9 = 8 × 9 × 100 = 72 × 100 = 7200.

And 90 × 9 is 9 × 9 × 10 = 81 × 10 = 810.

We also have that 1 × 9 = 9.

Therefore we have 891 × 9 = 7200 + 810 + 9 = 8019.

Now let's look at another method to calculate the same sort of product (multiplication).

Method 2

It will be useful for later topics to think of this product set out in a grid as follows. The number is split up in the same way and we multiply row by column.

×	800	90	1
9	7200	810	9

Then add up as before so that 891 × 9 = 7200 + 810 + 9 = 8019.

Method 3

Here we shall set it out in a similar way to how we did addition and subtraction. Suppose we wish to calculate 236 x 5. We proceed as follows.

$$\begin{array}{r} 2\ 3\ 6 \\ \times \qquad 5 \\ \hline 0 \\ \hline {\scriptstyle 3\ } \end{array}$$

Start by multiplying the ones; 5 × 6 = 30.
Fill in the '0' and carry the 3 tens so you can add them to the result of multiplying the tens.

$$\begin{array}{r} 2\ 3\ 6 \\ \times \qquad 5 \\ \hline 8\ 0 \\ \hline {\scriptstyle 1\ \ 3\ } \end{array}$$

Now do the tens. 5 × 3 tens = 15 tens, plus the 3 tens you carried, gives 18 tens. Fill in the 8 tens and carry the 1 (the '1' stands for 10 tens or 1 hundred).

$$\begin{array}{r} 2\ 3\ 6 \\ \times \qquad 5 \\ \hline 1\ 1\ 8\ 0 \\ \hline {\scriptstyle 1\ \ 3\ } \end{array}$$

Now do the hundreds. 5 × 2 hundreds = 10 hundreds, plus the one hundred you carried gives 11 hundreds, that is 1 thousand 1 hundred.

And so our answer is that 236 x 5 = 1180.

Activity 7

1 Find the following products without using a calculator.

(a) 34 × 7 (b) 54 × 3 (c) 48 × 8 (d) 91 × 4

Solve the following without using a calculator.

2 An agency worker earns £85 a day. How much do they earn over a five-day working week?

3 A teacher teaches three different classes the same material. Altogether, there are 73 students in the three classes. Normally when he asks for copies of a worksheet, he asks for 73 copies (he doesn't bother with spares). This week he wants each student to have two copies of a worksheet.

How many copies should he ask for?

4 Marion is doing a half-marathon to raise money for charity. She asks 66 people, and they each donate £5.

How much does she raise altogether?

5 An Indian restaurant, open seven days a week, gets through (on average) 47 onions a day.

How many onions do they get through in a week?

6 Find which one of the following numbers, when multiplied by 7, gives a product in which each of the digits is 3.

48619 47649 47719 47619

7 What is the ones digit of 211 × 213 × 217 × 219?

Now try these multiplications, using whatever method you prefer.

8 (a) 263 × 6 (b) 3078 × 9 (c) 49 970 × 8

9 A company employs two directors and six senior managers. The two directors are paid £65 402 a year. The senior managers each earn £50 064 a year.

How much money does the company spend on salaries for senior managers each year?

Why not make up more of your own and work them out? You can check your answers using your calculator.

Long multiplication – larger numbers

We have looked at multiplying a large number by a single-digit number. Next we shall look at multiplying two large numbers together (where both have more than one digit). We'll look at three different methods.

Method 1

Let's calculate 236 x 45 (or 45 x 236). We know that this is the same as 40 × 236 + 5 × 236. We just need a neat way to write this down and work it out. For this method, we'll write it out using the hundreds-tens-ones column approach. To work out 5 x 236 we've already seen that we can write this product as

$$\begin{array}{r} 2\ 3\ 6 \\ \times \quad\quad 5 \\ \hline \end{array}$$

So we can start by doing this calculation. This gives us 1180 as we have already seen.

$$
\begin{array}{rcccc}
 & & 2 & 3 & 6 \\
\times & & & & 5 \\
\hline
 & 1 & 1 & 8 & 0 \\
\hline
\end{array}
$$

Next we need to work out 236 by 40.

As we've seen, to multiply 236 by 40, we multiply by 4 and then by 10. However, since we know that the effect of multiplying by 10 is to move everything to the left by one column and putting a 0 in the ones column, we do that first:

$$
\begin{array}{rcccc}
 & & 2 & 3 & 6 \\
\times & & & 4 & 5 \\
\hline
 & 1 & 1 & 8 & 0 \\
 & & & & 0 \\
\hline
 & & & & \\
\hline
\end{array}
$$

So we've multiplied by 10, now we just need to multiply by 4, and we do this as before.

First multiply 4 by 6 to get 24. Next do the 4 times 3 which gives 12, and adding the 2 gives 14. Finally, 2 x 4 gives 8, and adding the 1 gives 9.

$$
\begin{array}{rcccc}
 & & 2 & 3 & 6 \\
\times & & & 4 & 5 \\
\hline
 & 1 & 1 & 8 & 0 \\
 & 9 & {}^{1}4 & {}^{2}4 & 0 \\
\hline
 & & & & \\
\hline
\end{array}
$$

So we have that 236 x 40 is 9440. Finally we need to add the 9440 to 1180. This method means they are already lined up in the correct columns, so this is quite straightforward.

$$
\begin{array}{rccccc}
 & & & 2 & 3 & 6 \\
\times & & & & 4 & 5 \\
\hline
 & & 1 & 1 & 8 & 0 \\
 & & 9 & 4 & 4 & 0 \\
\hline
 & 1 & 0 & 6 & 2 & 0 \\
\hline
\end{array}
$$

So we have worked out 236 × 40 and 236 × 5 and added them together to find that 236 × 45 = 10 620. You can check the answer on your calculator.

The next example shows the working for 478 × 63.

$$\begin{array}{r} 4\ 7\ 8 \\ \times \quad \underline{\quad\ 6\ 3} \\ 1\ 4\ 3\ 4 \\ \underline{2\ 8\ 6\ 8\ 0} \\ \underline{3\ 0\ 1\ 1\ 4} \end{array}$$

Method 2 – the Grid Method

Here is a different method for multiplying two large numbers, called the Grid Method. Let's multiply 478 x 63.

Again we use the idea that 478 × 63 = 478 × 60 + 478 × 3 and we then break each of these up further, so that 478 x 60 = 400 x 60 + 70 x 60 + 8 x 60, and similarly for the 478 x 3. To make it easy to see what's going on, we write it out in a grid.

We'll split up the two numbers into their hundreds, tens and ones 478 across the top, and the 63 down the side.

×	**400**	**70**	**8**
60			
3			

To fill in the grid, we multiply the two corresponding numbers.

First 400 x 60. Well 4 x 6 is 24, and adding on the three zeros gives us 24 000.

To do 70 x 60, 7 x 6 is 42 and adding two zeros gives us 4200.

Can you fill in the rest of the grid?

×	**400**	**70**	**8**
60	24000	4200	
3			

You should have got that

8 x 6 is 48, and adding the zero gives 480.

3 x 4 is 12 and adding the two zeros gives 1200.

3 x 7 is 21 and adding the zero gives 210.

And finally 3 x 8 is 24.

×	400	70	8
60	24000	4200	480
3	1200	210	24

All that remains to do is to add up the numbers in the grid.

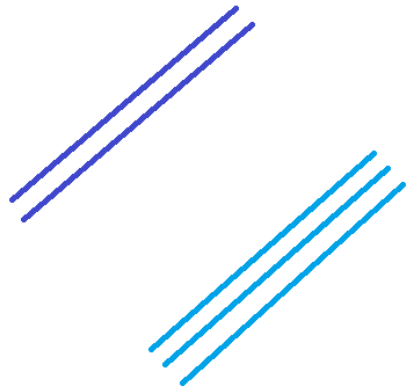
$$\begin{array}{r} 24000 \\ 4200 \\ 1200 \\ 480 \\ 210 \\ +\quad 24 \\ \hline 30114 \end{array}$$

So we've found that 478 x 63 = 30 114.

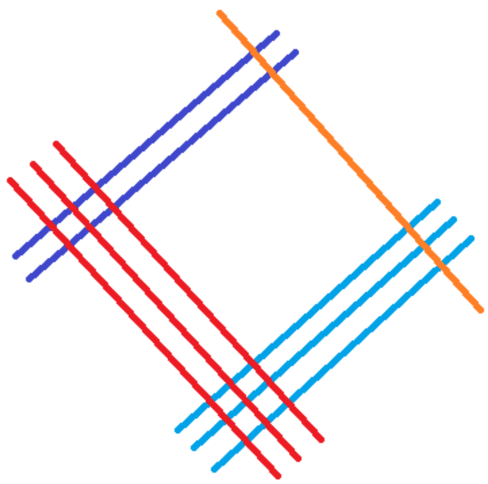
Method 3 – the Singapore Line Method

This final method is a more visual approach. It uses lines to represent numbers. We'll use it to calculate 23 x 31.

First we'll represent the number 23 using lines going diagonally up from left to right. Two lines to represent the 2 tens, and 3 lines to represent the 3 ones.



Next, we represent the number 31, this time using lines going diagonally down from left to right. Three lines to represent the 3 tens, and 1 line to represent the single 'one'.

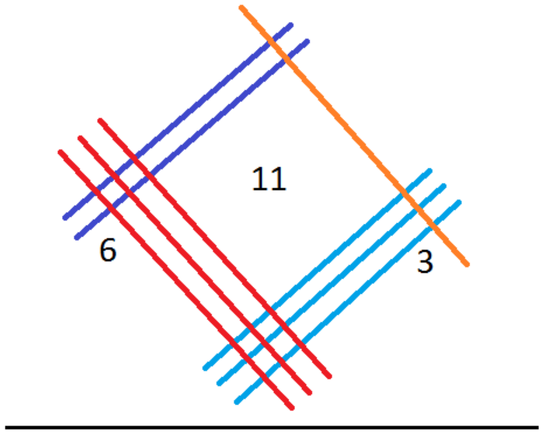


Now here's the clever bit. We add up the number of intersections of lines in the left-hand section, the middle section and the right-hand section.

In the left-hand section we have 6 intersections. This is the number of hundreds.

In the middle section we have two intersections at the top and 9 intersections at the bottom which makes 11 altogether. This is the number of tens.

Finally, in the right-hand section we have 3 intersections. This is the number of ones.

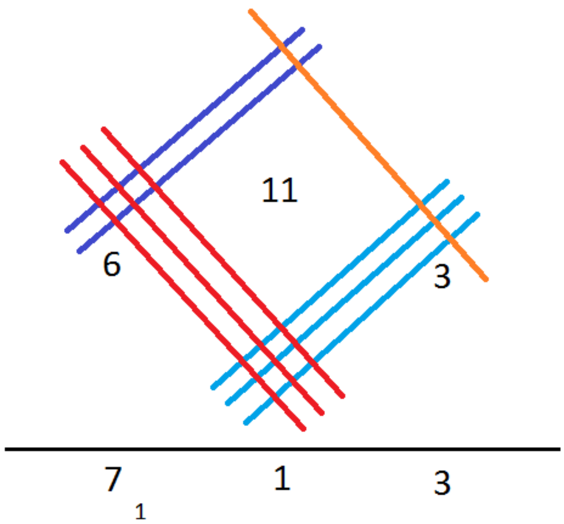


If all these numbers are under 10, then we would be able to read off the number immediately. In this case though, we have 11 tens, so we need to work as we would with columns.

In the ones column we have 3 ones.

In the tens column, we have 11 tens, so that's 1 ten in the tens column, and the 10 tens make 1 hundred which we carry over to the hundreds column.

Finally, in the hundreds column we have 6 hundreds, plus the 1 hundred that was carried makes 7 hundreds.



Therefore the solution is that 23 x 31 = 713.

Activity 8

1 Work out the following products without using a calculator. Choose whichever method you prefer, or perhaps try more than one to see which works better for you.

(a) 43 × 23 (b) 45 × 32 (c) 74 × 43 (d) 87 × 29

(e) 95 × 45 (f) 33 × 48 (g) 82 × 33 (h) 67 × 41

2 Work out the following products without using a calculator.

(a) 612 × 47 (b) 453 × 78 (c) 700 × 37 (d) 209 × 99

Division

Now we are going to look at division. Division is the process of seeing how many times one number will fit into another.

We'll look at two methods to do this – one is the more traditional way that you may have seen before, and the other is a method called 'chunking'.

Long division – dividing by a single-digit number

We are going to look at a method for dividing some number by a single digit number first of all, for example 332 ÷ 4.

If we wish to calculate 332 ÷ 4, then we shall write it in the following way:

$$4\overline{)332}$$

where again, the 332 is set out in columns for hundreds, tens and ones.

We start by dividing the first digit on the left-hand side of the large number (i.e. the digit in the hundreds column) by 4.

In this case, the first digit is 3 and 4 is bigger than 3 so we cannot divide 4 into it. So 4 divides into 3 zero times and we carry the 3 along to make the 3 in the tens column 33.

$$\begin{array}{l} \;\;\;0 \\ 4\overline{)3\,^{3}3\,2} \end{array}$$

When we divide 33 by 4, we can fit in 8 lots of 4 (which makes 32) and have 1 left over, so the result is 8 with a remainder of 1. We write down the 8 on top of the division sum and keep the remainder 1 which we write as a tens digit to the left of the next digit

$$\begin{array}{l} \;\;\;0\;\;8 \\ 4\overline{)3\,^{3}3\,^{1}2} \end{array}$$

The next step is to do 12 divide by 4, which luckily goes exactly as 3 which we write down next to the 8

$$\begin{array}{l} \;\;\;0\;\;8\;3 \\ 4\overline{)3\,^{3}3\,^{1}2} \end{array}$$

And so we've found that 332 ÷ 4 = 83.

Here is another example, 27703 ÷ 9.

$$9\overline{)27433}$$

We divide the 9 into the 2 first of all but 2 is smaller than 9 so we write a 0 in the ten thousand column and carry the 2.

$$\begin{array}{l} \;\;\;0 \\ 9\overline{)2\,^{2}7433} \end{array}$$

Now we have 27 ÷ 9, which is 3 exactly so the remainder is 0. So we write 3 on top and we don't put anything by the next digit.

$$\begin{array}{l} \;\;\;0\;\;3 \\ 9\overline{)2\,^{2}7433} \end{array}$$

Now we divide the 9 into the next digit, 4, but 4 is smaller than 9 and so *we write a zero* and have remainder 4, which we carry to onto the next digit, 3.

$$\begin{array}{r} 0\;\;30 \\ 9\overline{)2^{2}74^{4}33} \end{array}$$

Now we have 43 ÷ 9. We can fit 4 lots of 9 into 43 and have 7 left over, so, we write 4 next to the 0 and carry over the remainder of 7

$$\begin{array}{r} 0\;\;30\,4\;\; \\ 9\overline{)2^{2}74^{4}3^{7}3} \end{array}$$

Finally, we have 73 ÷ 9. We can fit 8 lots of 9 into 73, and we're left with a remainder of 1.

$$\begin{array}{r} 0\;\;30\,4\;\;8 \\ 9\overline{)2^{2}74^{4}3^{7}3} \end{array}$$

remainder 1

There's nothing we can do about the left over 1. And so, the solution to the problem of 27433 ÷ 9 is 3048, with a remainder of 1. That means that you can fit 3048 lots of 9 into 27433 and there will be 1 left over. If we wanted, we could check this with our calculator.

Activity 9

Carry out the following calculations without using a calculator.

(a) 3448 ÷ 8 (b) 8561 ÷ 7 (c) 45 890 ÷ 5

(d) A grandmother leaves assets of £47 538 in her will to be divided between her nine grandchildren. How much does each grandchild receive?

(e) A very large, open-plan office block has 569 248 square feet of office space, divided equally over four floors. How much office space is on each floor?

(f) 479 364 ÷ 6

When you have finished, check your answers on your calculator.

Long division – dividing by a larger number

Here we're looking at calculating larger numbers, for example

$$368 \div 16$$

where the number we're dividing by has more than one digit. This is harder because we can't use our times tables in the same way. We could use the same method as before, but it is trickier. Instead we shall use a method called Chunking.

Chunking is based on the idea that division can be thought of as seeing how many lots of one number will fit into another.

So calculating 368 ÷ 16 is the same as asking "How many lots of 16 will fit into 368?"

To find out the answer to this, we will take chunks of 16s off 368 (keeping track of how many we've taken) until we can't take any more. The number of 16s we've taken off will be our answer.

We could take off one 16 at a time, but this would take quite a while, so let's work out some simple multiples of 16 first:

1 x 16 = 16

2 x 16 = 32

10 x 16 = 160

5 x 16 = 80 (since this is half of 10 lots of 16)

20 x 16 = 320 (since this is 10 lots of 2 x 16)

We can now take off 'chunks' at a time, keeping track of how many we've taken off.

We look to see what the largest number of 16s is that we can subtract. We can see that we can take off 20 × 16 = 320. So we do the subtraction, and make a note of how many 16s we've taken off.

$$\begin{array}{r} 368 \\ -320 \\ \hline 48 \end{array} \qquad 20$$

Now we have 48 left. How many 16s can we subtract? We can take off 2 × 16 = 32.

$$\begin{array}{rl}
368 & \\
-320 & 20 \\
\hline
48 & \\
-\ 32 & 2 \\
\hline
16 &
\end{array}$$

Now we have 16 left. We can just subtract just one more 16.

$$\begin{array}{rl}
368 & \\
-320 & 20 \\
\hline
48 & \\
-\ 32 & 2 \\
\hline
16 & \\
-\ 16 & 1 \\
\hline
0 &
\end{array}$$

Now we have 0, so we obviously cannot subtract any more lots of 16 so we stop.

We can see that altogether we have managed to subtract 23 lots of 16 and so 23 16s will fit into 368.

That is 368 ÷ 16 = 23 with no remainder.

Feel free to use a different method if you prefer, just check with your tutor that it will always work.

Activity 10

Work out the following without using a calculator. Check your answer with your calculator when you have finished.

(a) 6375 ÷ 15 (b) 13608 ÷ 21 (c) 40953 ÷ 33

(d) 121400 ÷ 25 (e) 9290 ÷ 37 (f) 134911 ÷ 24

Order of operations

In the final section in this topic, we'll think about what to do if we have more than one operation in a calculation.

Below is a calculation with two different answers. Can you see how each answer was reached? Which is correct?

Calculation A: $4 \times 2 - 2 = 0$..

Calculation B: $4 \times 2 - 2 = 6$

Either could be correct!

Calculation A was done by working out the $2 - 2$ first, then multiplying by 4. We could write this as $4 \times (2 - 2) = 4 - 4 = 0$.

Calculation B was done by working out 4×2 first, then subtracting the 2. We could write this as $(4 \times 2) - 2 = 8 - 2 = 6$.

Two things stand out from this:

We need to have a rule for how we order the operations

We have used brackets to indicate which bit of the calculation should be done first.

As a consequence, we have the following rule, which orders the operations.

(Don't worry about this one for now, you will come to this later in the course.)

Brackets

Indices

Division and Multiplication

- do them in the order they appear, working from left to right

Addition and Subtraction

- do them in the order they appear, working from left to

It's sometimes known as BIDMAS which can help you remember the order.

It's important to note that there is no logical or mathematical reason for choosing this order of operations. It's just that, as we've seen, we need some sort of order – and this is it!

Example: Calculate 5 + 7 x 12 ÷ 4 – (5 + 7) using the BIDMAS rule.

Following the order of BIDMAS, we need to do the Brackets first:

$$5 + 7 \times 12 \div 4 - (5 + 7)$$

$$= 5 + 7 \times 12 \div 4 - 12$$

Next we do the Divisions and Multiplications in the order they appear, working from left to right:

$$5 + 7 \times 12 \div 4 - 12$$

$$= 5 + 84 \div 4 - 12$$

$$= 5 + 84 \div 4 - 12$$

$$= 5 + 21 - 12$$

Next we do the Additions and Subtractions in the order they appear, working from left to right:

$$5 + 21 - 12$$

$$= 26 - 12$$

$$= 26 - 12$$

$$= 14$$

Keypoint

When you have a calculation involving several operations, do brackets first, then division and multiplication (in the order they appear), then addition and subtraction (in the order they appear).

Activity 11

1 Work out the following:

(a) $4 \times 3 - 6$ (b) $10 \div 2 + 7$ (c) $9 + 4 \times 3$ (d) $2 + 6 \div 6$

(e) $2 \times 2 - 2$ (f) $7 - 4 \div (3 - 1)$ (g) $2 + 16 \div (2 x 4) x 2$

2 Are the calculations below correct?

(a) $8 - 3 \times 2 = 2$ (b) $(8 - 3) \times 2 = 2$

(c) $8 - (3 \times 2) = 2$ (d) $(5 + 3) \times 2 = 16$

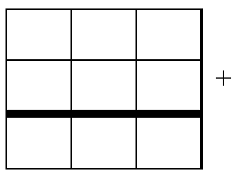
(e) $5 + 3 \times 2 = 16$ (f) $8 \div 4 - 2 = 4$

(g) $8 \div 4 - 2 = 0$ (h) $5 - 2 \times 3 + 2 = 11$

(i) $(5 - 2) \times 3 + 2 = 11$ (j) $5 - 2 \times (3 + 2) = 11$

Activity 12 – Problem solving and reasoning

1 Can you fill in the boxes so that the addition works and no digits are repeated?



2 Intending to multiply a number by 36, Lucas accidently multiplied it by 63 instead. The answer he got was 1566 too big. What was the number he was multiplying?

Key terms

Difference The result of subtracting one number from another

Digit A numeral between zero and nine, which means that, for example, 3 is a single-digit number and 68 is a two-digit number

Integer A whole number, including zero

Operation The instruction to do something with some numbers

Place value The value a **digit** has in a number due to its position in that number – e.g. the place value of 3 in 234 is 30

Positive integers A whole number which is greater than zero

Product The result of multiplying two numbers together, e.g. the product of 2 x 3 is 6

Sum The result of adding two numbers together, e.g. the sum of 2 + 3 is 5

Summary

After finishing this topic, you should be able to:

- recall the times tables up to 12 x 12
- find the complement to 10 of any single-digit number and the complement to 100 of any two-digit number
- add any two whole numbers without a calculator
- subtract any two whole numbers without a calculator
- multiply any number by a multiple of 10, e.g. 43 × 20
- multiply any number by a single digit without a calculator, e.g. 473 × 7
- multiply any two whole numbers together without a calculator, e.g. 37 × 64 and 45 × 271
- divide any number by a single digit, e.g. 873 ÷ 6 and appreciate that there may be a remainder
- divide any whole number by another whole number, e.g. 8734 ÷ 32
- define the words integer, sum, difference, product and operation
- apply the skills learned to solve real-world problems.

Answers

Activity 1

1 (a) 40 (b) 200 000 (c) 3 000

2 (a) seven hundred (b) nine

(c) four million (d) three hundred thousand

Activity 2

Table 1

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

(a) True

(b) False, for example $2 \times 5 = 10$ which does not end in 5.

(c) False, for example $2 \times 7 = 14$ and $1 + 4 = 5$.

(d) True, The only value that spoils the pattern is 99, since $9 + 9 = 18$.

However, you could then add up the two digits of this number to get

$1 + 8 = 9$

What about other, larger, multiples of 9...?

(e) True, what about for other, larger, multiples of 11...?

(f) False, as an example, 1 x 12 = 12 and 1 + 2 = 3 which is odd.

(g) True, called the square numbers.

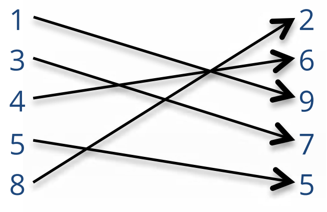
(h) True.

(i) False, As an example, 3 x 9 = 27 and the ones digit of 27 is '7'.

(j) True. This means that 6 x 4 = 4 x 6.

Activity 3

1 1 and 9, 2 and 8, 3 and 7, 4 and 6, 5 and 5



2	(a)	87	(b)	55	(c)	49	(d)	66
	(e)	34	(f)	9	(g)	72	(h)	93

Activity 4

(a) 599	(b) 989	(c) 794
(d) 943	(e) 1185	(f) 2150
(g) 1227 pupils	(h) 185 guests	

Activity 5

(a) 161	(b) 529	(c) 231
(d) 61	(e) 1103	

(f) 6423 fliers. The calculation that was needed was 6750 – 327. The 5000 houses was irrelevant information.

(g) £2142 – £250 = £1892, and so, again, the breakdown of the costs was irrelevant to the question.

(h) 78 129 – 77 893 = 236 miles

Activity 6

(a) 90	(b) 980	(c) 3 200	(d) 330
(e) 6 300	(f) 80	(g) 210	(h) 40 000
(i) 8 000	(j) 480 000	(k) 3 600	(l) 12 100

Activity 7

1 (a) 238 (b) 162 (c) 384 (d) 364

2 £425

3 146 worksheets

4 £330

5 329 onions

6 47619 × 7 = 333 333

7 Notice that you don't need to do the whole calculation, you can just multiply the ones digit in each of the four numbers together. The ones digit of this answer is 9. All the other multiplications for the other columns will involve multiples of ten, and so will not affect the ones column.

8 (a) 1578 (b) 27 702 (c) 399 760

9 £300 384

Activity 8

1 (a) 989 (b) 1440 (c) 3182 (d) 2523

(e) 4275 (f) 1584 (g) 2706 (h) 2747

2 (a) 28 764 (b) 35 334 (c) 25 900 (d) 20 691

With (d), you could also work it out by finding 209 × 100, which is 20 900, and then subtracting 209 to give the answer for 209 × 99.

Activity 9

(a) 431 (b) 1223 (c) 9178

(d) £5 282 (e) 142 312 square feet

(f) 79 894

Activity 10

(a) 425 (b) 648 (c) 1241

(d) 4856 (e) 251 rem 3 (f) 5621 rem 7

Activity 11

1 (a) 6 (b) 12 (c) 21
(d) 3 (e) 2 (f) 5
(g) 6

2 (a) correct (b) incorrect, the correct answer is 10
(c) correct. Note that these brackets weren't really needed here, as you would do the multiplication first anyway.
(d) correct (e) incorrect, the correct answer is 11
(f) incorrect, the correct answer is 0
(g) correct (h) incorrect, the correct answer is 1
(i) correct
(j) incorrect, the correct answer is –5. (You'll meet negative numbers in a later topic.)

Activity 12 – Problem solving and reasoning

1 Here is one solution, there may be others!

7	8	2	
1	6	3	+
9	4	5	

2 The number was 58. Since 63 is 27 more than 36, Lucas multiplied his number by 27 too many. So 27 lots of Lucas's number was 1566. 1566 ÷ 27 = 58 so Lucas's number was 58.

What next?

We hope this sample has helped you to decide whether this course is right for you.

If you have any further questions, please do not hesitate to contact us using the details below.

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