NATIONAL EXTENSION COLLEGE

# Structured Fast Track

# A level Maths

## Course sample

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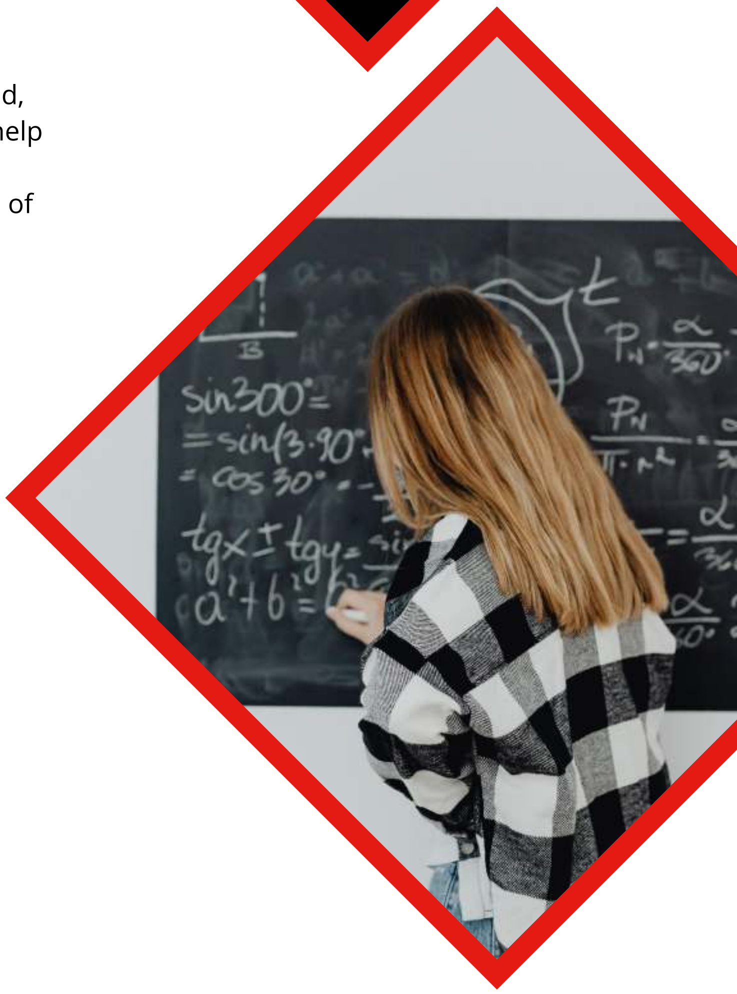

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# So what will a course topic look like?

# Course content

NEC's Structured Fast Track A level Maths course will follow the same topics as Pearson Edexcel's maths A level but has a set schedule, allowing you to complete the course within one academic year rather than two.

**Section 1: Pure mathematics** 1

Algebra 1

**Let's look at part of 'Topic 1 Algebra 1'**

Indices and surds

Coordinate geometry

Quadratics

Graphs

Trigonometry 1

**Section 2: Pure mathematics 2**

Algebra 2

Circles

The binomial expansion

Differentiation 1

Exponentials and logarithms 1

**Section 3: Pure mathematics 3**

Trigonometry 2

Differentiation 2

Exponentials and logarithms 2

Integration

Vectors

**Section 4: Statistics 1**

Mathematical models in statistics

Measures of location and spread

Representing data

Probability

Hypothesis testing

**Section 5: Mechanics 1**

Mathematical models in mechanics

Variable acceleration

**Section 6: Pure Mathematics 4**

Algebra 3

Functions and graphs

Trigonometry 3

Sequences and series

**Section 7: Pure Mathematics 5**

Trigonometry 4

Sequences and series 2

Differentiation 3

Integration 2

**Section 8: Pure Mathematics 6**

Numerical methods

Differentiation 4

Parameters

Integration 3

Differential equations

**Section 9 – Statistics 2**

Correlation and regression

Probability

The normal distribution

Approximations and testing

**Section 10 – Mechanics 2**

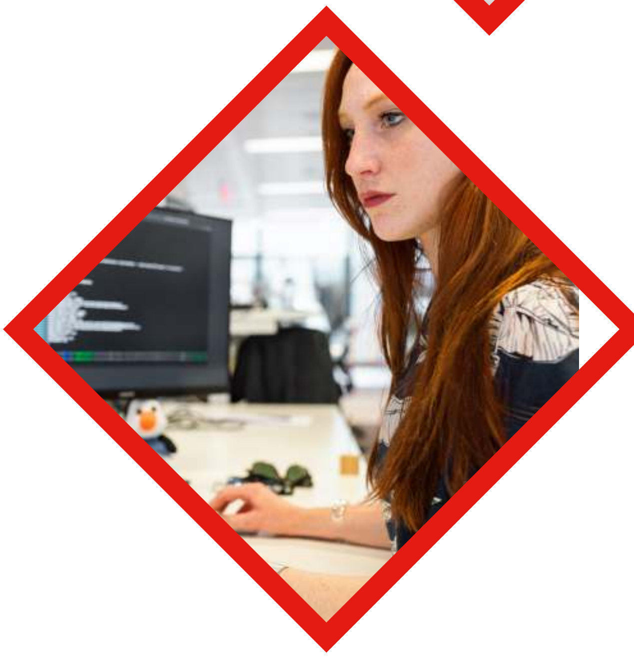
Moments

Equilibrium 2

Forces and motion

Projectiles

Further kinematics



Section 1 Topic 1

# Algebra

## 1.1.1 Introduction

Algebra is the fundamental skill that you really have to acquire if your study of A level mathematics is to be successful. You have to keep up a high level of accuracy in completing fairly routine tasks. This can only come with plenty of practice, so there are many questions for you to do in this topic, and, of course, answers at the end of the topic so that you can check that your working is correct.

The basic techniques include **expanding brackets** and the reverse of this, **factorising**: this is what we look at first, followed by methods for more advanced factorisation. We then look in more detail at one particular technique that will be useful in the next topics – rewriting a **quadratic equation** so that we can read off the minimum or maximum point.

### Objectives

When you have completed this topic, you should be able to:

- understand what a polynomial is
- add, subtract and multiply polynomials
- use the notation $f(x)$, $g(x)$, etc. as a short way of writing polynomials
- take out a common factor
- factorise quadratics
- use the factor theorem to find linear factors of other polynomials
- divide polynomials by a linear factor.

# 1.1.2 Mathematical skills

You should have already picked up most of the basic skills required in algebra from your study of GCSE – it's now a question of applying those skills to longer questions and more complicated expressions.

In particular you need to be able to:

- remove brackets, taking care of any minus signs, e.g.

$$-3(x^2 - 2) = -3x^2 + 6$$

- multiply out brackets, e.g.

$$(x - 3)(2x^2 + 4) = 2x^3 - 6x^2 + 4x - 12$$

- manipulate fractions confidently, e.g.

$$\frac{2}{3} - \frac{1}{2} = \frac{4-3}{6} = \frac{1}{6}$$

- take out a common factor, e.g.

$$2x - 4x^2 = 2x(1 - 2x)$$

- factorise a quadratic expression, e.g.

$$x^2 - 4x - 12 = (x - 6)(x + 2)$$

As you progress through the topic you will find that the expressions become a little more involved – but at the same time your skills increase so that if you master the fundamentals now you will not find much to fear in the future.

# 1.1.3 Polynomials

Algebra uses letters which can take a variety of values instead of just one and for this reason are called **variables**.

A **polynomial** is a collection of terms involving a variable, usually $x$, such as $x^3 - 2x^2 + 5$, or simply $3x + 1$.

When the highest power of the variable is 2, for example $x^2 - 3x + 7$, it is called a **quadratic polynomial**, or quadratic for short.

A **cubic polynomial** such as $x^3 - 4x^2 + 5x + 4$ has a highest power of 3. It can also be called a polynomial of *order 3*.

A polynomial where the highest power is 1, such as $3x + 1$ or $x + 7$, is called a linear polynomial (often just described simply as being linear).

As you can see polynomials are usually written with the term involving the highest power first, then the powers go down until the term without the variable, called the **constant term**.

The numbers in the polynomial expressions are called **coefficients**. For example, in the polynomial $x^3 - 4x^2 + 5x + 4$, the coefficient of the $x^2$-term is $-4$ and the coefficient of the $x^3$-term is 1.

In fact, all the powers of the variable have to be positive integers (not counting the constant term).

Neither $x^3 + 2x^2 + \sqrt{x} - 3$ nor $x^2 + x - 3 + \dfrac{4}{x}$, for example, is a polynomial.

Think of these polynomials as functions in which we can input a value of $x$ and find an output. We can use function notation to label them by giving them a letter and writing $f(x)$ or $g(x)$ if our variable is $x$, so that we could write our cubic polynomial above as

$$f(x) = x^3 - 4x^2 + 5x + 4$$

We can find the value of the function for a particular value of the variable, for example.

The variable is not always $x$ – it can sometimes be another letter, $t$ say, in which case our cubic would be written as

$$f(t) = t^3 - 4t^2 + 5t + 4$$

Once we've labelled the polynomial like this, we can simply write $f(x)$ instead of writing it down in full when we need to refer to it. Now we can take another polynomial, our quadratic above for example, and call it $g(x)$, so that

$$g(x) = x^2 - 3x + 7$$

## Adding and subtracting polynomials

If we want to add these polynomials $f(x) + g(x)$, we add the like terms together, so that the $x^2$ terms are added together, and so on. For the moment, we can set this sum out as though we were adding numbers, although normally you would write them on one line:

$$\begin{array}{lr} f(x) & x^3 - 4x^2 + 5x + 4 \\ +\ g(x) & x^2 - 3x + 7 \\ \hline & x^3 - 3x^2 + 2x + 11 \end{array}$$

We use the same method of combining like terms when it comes to subtracting polynomials, but we have to be *very* careful to watch the signs. Probably the best way until you are very confident is to take the subtraction in stages.

Suppose you want to subtract $g(x)$ from $f(x)$. You could write

$$f(x) - g(x) = (x^3 - 4x^2 + 5x + 4) - (x^2 - 3x + 7)$$

Since there is nothing outside the first bracket on the right-hand side we don't need it and we can write the polynomial without it.

The last bracket has a negative sign outside and this means that *all* the signs have to be changed if we want to remove it, because we are subtracting every term in the second bracket.

When this has been done, we add the like terms together as we did when we added the polynomials.

$$\begin{aligned} f(x) - g(x) &= x^3 - 4x^2 + 5x + 4 - x^2 + 3x - 7 \\ &= x^3 - 5x^2 + 8x - 3 \end{aligned}$$

## Multiplying polynomials

You probably already know that to multiply out a bracket such as $2x(3x - 4)$, you multiply both terms inside the bracket by the $2x$, to give

$$2x(3x - 4) = 6x^2 - 8x$$

To expand an expression such as $(2x + 3)(3x - 4)$ we split the first bracket into two parts and multiply out each separately, collecting together any like terms:

$$\begin{aligned} (2x + 3)(3x - 4) &= 2x(3x - 4) + 3(3x - 4) \\ &= 6x^2 - 8x + 9x - 12 \\ &= 6x^2 + x - 12 \end{aligned}$$

When multiplying more complicated polynomials, you need to be quite systematic. Suppose we had to find the product of two quadratics, for example:

$$(1 - 3x + x^2)(1 - 4x - x^2)$$

One way would be to multiply everything in the right-hand bracket by each of the terms in the left-hand bracket in turn, and then collect like terms together:

$$\begin{aligned} (1 - 3x + x^2)(1 - 4x - x^2) &= 1(1 - 4x - x^2) - 3x(1 - 4x - x^2) + x^2(1 - 4x - x^2) \\ &= 1 - 4x - x^2 - 3x + 12x^2 + 3x^3 + x^2 - 4x^3 - x^4 \\ &= 1 - 7x + 12x^2 - x^3 - x^4 \end{aligned}$$

Make sure that you can multiply out the polynomials in the following practice questions without making mistakes. If you do make a mistake, try to see what the product was that led to the slip and correct it.

As you work through this topic and complete further practice exercises, you should begin to get a firm basic knowledge of the complicated rules for combining and separating these polynomials. Some of the ideas may already be familiar to you, but there will be others that are probably new.

Now do the following questions.

## Practice questions

1 Expand:

(a) $3x(2x + 4)$ (b) $3(2x - 1)$

(c) $-4(2x - 3)$ (d) $-2x(1 + x)$

(e) $3x(1 - 3x)$ (f) $-4x(5 - 3x)$

2 Expand:

(a) $(4x + 1)(2x - 3)$ (b) $(2x - 1)(3x - 2)$

(c) $(2x - 3)(2x + 3)$ (d) $(1 - x)(1 + 2x)$

(e) $(1 + 3x)(2 - x)$ (f) $(1 + 4x)(3x - 2)$

3 Expand:

(a) $3x(1 - 2x + x^2)$ (b) $(1 - 2x)(1 + x + 2x^2)$

(c) $(1 - x)(1 + x + x^2)$ (d) $(1 - 2x + x^2)(1 + 2x - x^2)$

(e) $(1 - x - x^2)(3 + 2x + 3x^2)$

(f) $(1 + x + x^2)^2$

4 If

$f(x) = 3x^3 - 2x^2 + 5x - 7$

$g(x) = 4x^2 - 3x + 2$

$h(x) = 2x - 1$

find

(a) $f(x) + g(x)$ (b) $f(x) - g(x)$

(c) $g(x) + 2h(x)$ (d) $f(x) - 3h(x)$

(e) $2g(x) - f(x)$ (f) $g(x) \times h(x)$

Answers are provided at the end of this topic.

# 1.1.4 Factorising

Factorising is an essential skill to acquire. In general, students who are able to factorise reasonably easily find the algebra in this topic quite manageable.

## Terms with a common factor

These are the easiest type, where you can spot a common factor in each of the two terms.

## Example

Factorise $3x^2 - 12x$.

## Solution

1 Find the highest number that will divide both terms: here 3.

2 Find the highest power of the variable dividing both terms: here $x$.

3 Combine these two outside a bracket: here $3x(\ )$.

4 Inside the bracket find the terms that give the original expression: here $3x(x - 4)$

## Practice questions

5 Factorise

| | | |
|---|---|---|
| (a) $3x - 6x^2$ | (b) $4xy + 8y$ | (c) $pq^3 - p^2q$ |
| (d) $12x^2 + 16x^3$ | (e) $6p - 9q$ | (f) $x + 5x^2$ |

# Factorising a quadratic

Suppose we have to factorise $x^2 + 7x + 12$, which mean expressing it in the form $(x + a)(x + b)$, where $a$ and $b$ are numbers we have to find.

If we multiply these brackets out, giving $x^2 + (a + b)x + ab$ and compare the quadratics, we can see that we want to choose $a$ and $b$ so that $ab = 12$ and $a + b = 7$.

This is quite easy: one must be 3, the other 4 and so the factors are $(x + 3)(x + 4)$.

Note that if the original expression had been $x^2 - 7x + 12$, we would still have $ab = 12$ but now $a + b = -7$. Since the product $ab$ is positive, the numbers $a$ and $b$ have the same signs; since the sum $a + b$ is negative, they must both be negative and so the factors in this case would be $(x - 3)(x - 4)$.

If $ab$ is negative, then $a$ and $b$ have opposite signs. In this case we have to look at the sum $a + b$, the $x$-coefficient. If this is positive, then the larger of $a$ and $b$ is positive: if negative, then the larger is negative.

So for example with $x^2 + 2x - 8$: $ab = -8$ means that $a$ and $b$ have opposite signs.

$a + b = 2$, so the larger is positive: $+4$, $-2$ with factors $(x + 4)(x - 2)$.

If instead we have $x^2 - x - 12$: $ab = -12$, again opposite signs but this time the larger is negative: $-4$, $+3$ with factors $(x - 4)(x + 3)$.

You may already know that if the $x$-coefficient is zero, for example $x^2 - 16$, the numbers $a$ and $b$ are the same but have opposite signs; here 4 and $-4$ with factors $(x + 4)(x - 4)$. This type of expression is called the **difference between two squares**.

You can always check you have the right factorised form: multiply your brackets out and make sure you get the quadratic expression you were given.

## Example

Factorise the following:

(a) $x^2 + 4x + 3$

(b) $x^2 - 6x + 8$

(c) $x^2 + x - 12$

(d) $x^2 - 3x - 10$

(e) $x^2 - 4$

## Solution

(a) Here $ab = 3, a + b = 4 \Rightarrow$ both positive;

1 and 3 $\Rightarrow (x + 1)(x + 3)$

**Tip**

'$\Rightarrow$' is the implication sign: a $\Rightarrow$ b means a implies b. You will find it cropping up everywhere – usually you can translate it as meaning 'so', e.g. $ab = 3$, $a + b = 4$, so both positive.

(b) Here $ab = 8, a + b = -6$ so both negative.

$-2$ and $-4 \Rightarrow (x - 2)(x - 4)$

(c) Here $ab = -12, a + b = 1$ so opposite signs, and positive bigger;
4 and $-3 \Rightarrow (x + 4)(x - 3)$

(d) Here $ab = -10, a + b = -3$ so opposite signs, negative bigger;
$-5$ and 2 $\Rightarrow (x - 5)(x + 2)$

(e) Here $ab = -4$ and $a + b = 0 \Rightarrow$ equal and opposite;

2 and $-2 \Rightarrow (x + 2)(x - 2)$

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